Project Einstein: Abstracts and Paper Presentation
Number 1 (92-97)
Journal homepage: http://www.dlsu.edu.ph/conferences/project-einstein
Project Einstein
ISSN 2467-785X

# Observer-Emergent Time, Space and Inertial Laws

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Received 15 March 2016 ; received in revised form 30 March 2016 ; accepted 13 April 2016

***Abstract*—The role of the observer in physics is reconsidered in connection with our evolving assumptions about the nature of time. Unlike the observer's optional movement through space, evolution in time is compulsory and strictly governed by causal order. Thus, it is argued that time is intrinsically more fundamental than space. The geometry of space is appreciated as emergent degrees of freedom allowing observers to develop invariant transformation equations between them. Inertial laws are shown to emerge from the observer's assumptions about the nature of time and its transformations. This leads us to explore the next logical extension of the observer's progressing appreciation of the nature of time by allowing time to depend on higher-derivative kinematical quantities. This leads to inertial laws that may potentially accommodate various nonlinear phenomena such as the yet unresolved celestial anomalies, and possibly provide the needed degrees of freedom for the unification of classical with quantum theory.**

*Keywords— observer; emergent manifold; causality; time dilation; relativity of time; higher-derivative theories; modified inertia; alternative gravity theory component*

This work was partially supported by De La Salle University – Manila.



## I. THE OBSERVER IN PHYSICS

Science negotiates its way through a constantly constricting spiral of observations, formulation of hypotheses from the gathered data, and empirical testing of the hypotheses through experiments that allow even more accurate observations. The prime mover of this quest is the curious 'observer' which usually personifies collectively all past and present investigators participating in the pursuit for greater understanding of the physical phenomena.

### A. *The Observer and Measurement*

Physics in particular revolves around measurement. Crucial to the exercise of measurement is the existence of some conscious intelligent being capable of performing, recording and possibly analyzing the measured quantities. Paradoxically, this entity also consists of measurable elements, being itself part of the universe that it is aiming to comprehend. So in a very profound way, physics is a manifestation of the universe attempting to understand itself. To be self-consistent, the universe must be comprehensible to itself, a mystery that astounded even Einstein's brilliant mind.

From the observer's collection of measured quantities, such as time intervals, space intervals, amount of matter, electric charge, magnetic moments, and so forth, the observer deduces some working theory that convincingly makes sense of the measurements. Then to explore the limits of the theory's validity, further measurements using more refined experiments are conducted which hopefully leads one to a more general theory. This exercise is repeated in the hope that an ultimate theory that explains all measurements in any realm and scale of the universe can be found.

Thus, we see that the scientific method is anchored upon how the observer appreciates physical phenomena. At any stage of inquiry, the observer endeavours to comprehend the universe through the discovered fundamental laws that govern the dynamics of its elements. The observer employs the appropriate inertial law depending on the scale and nature of the realm under investigation. For instance, one uses Newton's law of inertia for macroscopic systems that move much slower than the speed of light, while Einstein's relativity is used for objects that may move close to light's speed.

### B. *Anomalous Measurements*

The keen observer will inevitably become perplexed by anomalies in measurements and these will inspire modifications of conventional laws. A classic case is the observation that after a planet fly–by, satellites gain an orbit speed faster than what is expected from Newtonian dynamics. This fly-by anomaly[1] was observed from the Deep Space Network (DSN) Doppler data after the Earth-flyby of the Galileo in 1990 registering a 3.92 mm/s speed increase at its perigee. The Near Earth Asteroid Rendezvous (NEAR) spacecraft also registered an increase of 13.46 mm/s after its Earth fly-by in 1998. The Cassini-Huygens gained about 0.11

mm/s in 1999 and Rosetta 1.82 mm/s in 2005. Suspected to be related to this is the extra Sun-ward acceleration that Pioneer space crafts registered beyond 30AU [2,3,4,5]. Another anomalous observation is the early arrival of comets by a few days than what Newtonian dynamics expects. This advance has been shown to be possible if one assumes that farther than about 20AU from the Sun, an un-modelled acceleration of the order of Pioneer anomalous acceleration begins to take effect [6]. Granting such anomalies might be explained by sources of noise or engineering and design, they may nonetheless point to some needed fine-tunings in the conventional inertial law at very small accelerations.

The rotation of galaxies also appear to deviate from the Keplerian expectations, and this has compelled cosmologists to posit colossal amounts of unknown forms of matter that purportedly engulfs the galaxies, while others propose to modify the conventional laws of inertia or the gravitational field [7,8]. Cosmological data seem to be consistent with a universe filled with about 24% "dark matter" and 72% "dark energy" [9]. Many believe that this dark sector will eventually be experimentally detected, and Einstein's general relativity (GR) will be found to be consistent with experiments. Nevertheless, for as long as the experimental search comes short of the enormous amounts of non-baryonic matter required to account for the observed galactic rotations, one should continue to entertain alternative theories or extensions of general relativity.

As the observer begins to investigate subatomic experiments, the observer is forced to abandon the deterministic classical laws and embrace the probabilistic rules of quantum physics. Unlike general relativity which is a local classical theory, quantum mechanics has been known to exhibit non-locality [10]. Efforts to quantize gravity (Kaluza- Klein/string theories) invariably demand extra degrees of freedom and fields to mediate the gravitational interaction.

We see that our observer is beleaguered by incongruent treatments of the universe at various scales. Hence, it is but natural to aspire for one 'parent' inertial law that entirely accommodates all realms of inquiry. This propensity of Nature to withhold her secrets motivates the observer to review how our assumptions about the nature of time lead to inertial laws, and then explore the possibility of allowing time to depend on higher-derivative kinematic quantities in the hope that the resulting inertial laws will provide the required degrees of freedom to explain the anomalies and help reveal the long sought-for unification of classical theory with quantum theory.

In the next section, we briefly discuss the role of time and the observer in physics and demonstrate how our operational notion of time dictates the law of inertia that governs the dynamics of objects. Thereafter, we shall focus on a geometrical approach to higher-derivative field equations.

## II. THE OBSERVER AND CAUSAL ORDER

Amongst all quantities accessible to measurement, time is primordial in the sense that without it, the observer cannot perform (or even plan to perform) any measurement, lacking the requisite degree of freedom to animate itself or at least, its thought-process. Moreover, the observer progresses through time whether it chooses or not; but in contrast, the observer may decide not to evolve in space by simply maintaining its current state of rest or uniform motion. Clearly, evolution in time is compulsory, but movement in space is optional, at least classically. Furthermore, one can only evolve in time in a way that preserves causality. This non-commutative ordering of points in time is not required for positions in space. Notwithstanding the relativistic custom of treating space and time at equal footing, time is evidently more fundamental than space, especially to the observer. In quantum field theory for instance, a time-ordering operator is employed in the Dyson series, but no such feat is performed for space. One of the more promising approaches to quantum gravity known as causal dynamical triangulation (CDT) [11], even assumes no pre-existing arena (dimensional space). Instead, CDT demonstrates how the spacetime fabric itself emanates from time and causal order. Its proponents conjecture that, unlike space, time is truly fundamental and not merely *emergent*—a term that refers to some large scale or low-energy phenomenon. They view time as a truly intrinsic parameter with neither a beginning nor end; time and causal order persist on the very smallest scales (Planck scale). In contrast, the circumstance that space is three-dimensional and extended is appreciated as an *emergent* phenomenon. Time existed even before the big bang when space and its geometry have yet to manifest from the transformations that shall maintain order and consistency between measurements of observers.

The depth of treatment of the dynamics of a system depends on the observer's assumptions about the nature of time together with the amount of detail that these shall entail in the theoretical exercise. Time can be thought of as a perception of change along with the causal ordering of events which in physics must all be consistently relatable between observers. An observer at rest in a certain frame of reference shall be regarded as a *proper observer* in that frame of reference. A proper observer measures proper quantities such as rest mass, rest length and proper time intervals between events in its rest frame. To a *solipsist*, proper measurements would be all that would matter. But science normally consents to the existence of other observers as evidenced by their faithful reliance on related literature detailing the experimental results and conclusions of other observers. Consistency demands that the emergent space (along with its geometrical properties) between observers, come with invariant transformation equations relating the measurements of observers in diverse states of motion. Any other observer who is in motion relative to the proper observer shall be referred to as an "improper observer" in that frame of reference. The *improper observer* must subscribe to the universal transformation equations in order to make sense of the measurements of the proper observer, and vice versa.

### A. *The observer's assumptions about the nature of time*

We demonstrate in this section that within any realm of inquiry, the observer's operational assumptions about the nature of time delimit the law of inertia that governs the dynamics of systems. In retrospect, pinning down the intricacies of time in some mathematical form has been the preoccupation of theoretical physicists since Galileo mathematized his observations of natural phenomena. Galileo implicitly assumed an observer-independent time. Using this absolute time assumption, Sir Isaac Newton revealed the law that explains the dynamics of macroscopic objects as long as they move at speeds much slower than the speed of light. Einstein, however realized that time is only apparently absolute or independent of the motion of the observer when speeds are negligibly slower than that of light, and in 1905 elevated Newton's dynamics to include objects traveling with speeds nearing that of light. In Einstein's special theory of relativity, time acquires a velocity-dependence, and the observer realizes the inseparability of time and space. Henceforth, the universe has been attributed a spacetime manifold that preserves the constancy of the speed of light relative to all observers.

Einstein completed his general theory of relativity in 1916 in which the geometry of the spacetime governs the dynamics of objects, whilst the distribution of matter determines the geometry of the spacetime [12]. Subsequently, in general relativity, time depends on position and velocity. Thus, it would be reasonable to suspect that even more general laws might follow if time is allowed to depend on higher- derivative kinematical quantities. We find that this paradigm not only recovers the familiar classical theories through their respective time ansatz, but also paves the way to higher derivative extensions of inertial laws.

### *B. A time-centred paradigm*

To demonstrate our time-centered paradigm, let us consider some classical theory in which the linear momentum of a particle of rest mass $m$ and position $x_i(\tau)$, $i = 1,2,3$ is

$$p_i = m\frac{dx_i}{d\tau} = m\frac{dt}{d\tau}\frac{dx_i}{dt} = m\frac{dt}{d\tau}v_i \quad (1)$$

where $\tau$ is the proper observer's measurement of time in its rest frame, while $t$ is the time measured in some other reference frame which may be in motion relative to the first. We may refer to $t$ as the improper or coordinate time. The law of inertia specifies how the linear momentum varies in time,

$$\frac{dp_i}{dt} = m\frac{dt}{d\tau}a_i + m\left(\frac{dt}{d\tau}\right)^{-1}\frac{d^2t}{d\tau^2}v_i \quad (2)$$

where the velocity and acceleration are respectively,

$$v_i \equiv \frac{dx_i}{dt}, \qquad a_i \equiv \frac{dv_i}{dt} \quad (3)$$

The law of inertia (2) and the subsequent concept of "force" owe their ultimate forms on one's assumption about how the proper and improper times transform to each other.

The Galilean concept of universal time assumes that

$$\frac{dt}{d\tau} = 1. \quad (4)$$

The usual Newtonian law of inertia then follows from (2),

$$\frac{dp_i}{dt} = ma_i \quad (5)$$

where the force is always in the same direction as the acceleration.

If one assumes that the proper time $\tau$ and the coordinate time t are related by the relativistic time dilation equation

$$\frac{dt}{d\tau} \equiv \gamma(v) = \sqrt{1 - \frac{v_i v^i}{c^2}}, \quad (6)$$

then one finds from (2) the law of inertia in Einstein's special theory of relativity.[12]

$$\frac{dp_i}{dt} = m\gamma\left(\delta_{ij} + \frac{1}{c^2}\gamma^2(v)v_i v_j\right)a^j \quad (7)$$

The observer sees that the special theory of relativity follows from a velocity-dependent time ansatz (6) in consonance with Lorentz invariance.

In the attempt to explain the rotational curves of galaxies without requiring dark matter, people have seriously incorporated acceleration in the time transformation in regions of extremely low accelerations [7,8].

$$\frac{dt}{d\tau} = \sqrt{\mu(\alpha)} \quad (8)$$

where the function $\mu(\alpha)$ has asymptotic properties:

$$\mu(\alpha) \to \begin{cases} 1 \ \text{for}\ \alpha \equiv \sqrt{a_i a^i}/a_c \gg 1 \\ \alpha \ \text{for} \qquad \alpha \ll 1 \end{cases} \quad (9)$$

but otherwise arbitrary. In this theory, $a_c$ is some characteristic acceleration that marks the asymptotic region between Newtonian and modified inertial laws. With the time ansatz (8), the law of inertia that follows from (2) is [8]

$$\frac{dp_i}{dt} = m\sqrt{\mu}\,a_i + \frac{m}{2a_c\alpha\sqrt{\mu}}\frac{d\mu}{d\alpha}\,a_k j_k\,v_i \quad (10)$$

which involves the jerk

$$j_k \equiv \frac{da_k}{dt} \quad (11)$$

The corresponding expression for the conserved energy is [8]

$$E = \frac{1}{2}m\mu(\alpha)v_i v^i + U(x_i) \quad (12)$$

where $U$ is some external conservative field. The inertial law (10) is non-relativistic, but surprisingly, it reproduces the rotational dynamics of galaxies without requiring dark matter [7,8]. We note that the presence of the higher-derivative $d^2t/d\tau^2$ in (2), implies that in general, the "force" $dp/dt$ may not be in the same direction as the acceleration, as seen for instance in (7) and (10).

### *C. A time portal to a higher-derivative law of inertia*

As a natural extension of the inertial laws, we explore the possibility of allowing the transformation between the coordinate time $t$ and the proper time $t$ to involve any order of derivatives of the position $x_i(\tau)$:

$$\frac{dt}{d\tau} \equiv \tilde{\gamma}(x_i, v_i, a_i, j_i, \ldots) \quad (13)$$

Because a plethora of derivatives might lurk within the time transformation (13), we restore order overall these gradients by demanding Lorentz invariance:

$$E^2 - c^2 p_i p^i = m^2c^4. \quad (14)$$

The resulting energy-momentum 4-vector incorporates higher kinematical degrees of freedom

$$(E/c, p_i) = \left(mc\sqrt{1+\frac{1}{c^2}\tilde{\gamma}^2 v_k v^k}, m\tilde{\gamma}\, v_i\right) \quad (15)$$

From the time derivative of the energy,

$$\frac{dE}{dt} = \frac{\tilde{\gamma} v_i}{\sqrt{1+\frac{1}{c^2}\tilde{\gamma}^2 v_k v^k}}\frac{dp^i}{dt} \quad (16)$$

one finds that this energy is conserved if the "force" $d\mathbf{p}/dt$ is zero or at least perpendicular to the velocity **v**. The spacetime interval four-vector is evident from (15):

$$d\overleftrightarrow{s} \equiv \left(\frac{1}{mc}E d\tau, \frac{1}{m}p^i d\tau\right) = \left(c d\tau\sqrt{1+\frac{1}{c^2}\tilde{\gamma}^2 v_k v^k},\ dx_i\right) \quad (17)$$

The square of the invariant interval,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = c^2 d\tau^2 \quad (18)$$

may be viewed as possessing a metric whose time component incorporates higher-derivative kinematical parameters

$$g_{00} = \frac{1}{c^2}v_k v^k + \tilde{\gamma}^{-2} \quad (19)$$

The higher-derivative metric $g_{\mu\nu}$ naturally reduces to the flat Minkowskian metric $\eta_{\mu\nu}$ when $\tilde{\gamma} \to \gamma$.

By allowing our time transformation (13) to depend on higher-derivatives, we saw in (19) how space and its time-derivatives might emerge in the time-component $g_{00}$ of the metric even when the space sector is flat. But the space sector might be curved in general and may also have its higher-derivative dependencies. So we shall work within the assumption that through (13), the metric, assumed to be symmetric and invertible, can be formally cast so that it exhibits its *non-degenerate dependence* on the first $N^{th}$ derivatives of $x^\mu(\tau)$ with respect to the invariant proper time $\tau$:

$$\begin{aligned} &g_{\mu\nu} = g_{\mu\nu}(d^n x^\rho/d\tau^n),\\ &g_{\mu\nu} = g_{\nu\mu},\\ &g^{\sigma\rho}g_{\rho\mu} = \delta^\sigma_\mu\\ &n = 0,1,\ldots,N, \end{aligned} \quad (20)$$

Non-degeneracy here simply means that the presence of the highest-derivative $d^N x^\rho/d\tau^N$ in the invariant interval,

$$\int_{s_1}^{s_2} ds = \int_{\tau_1}^{\tau_2} d\tau\sqrt{g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}} \quad (21)$$

cannot be removed by integration by parts. From the extremum of the invariant interval,

$$\delta\int_{s_1}^{s_2} ds = 0 \quad (22)$$

one finds a higher-derivative geodesic equation

$$\frac{d^2x^\sigma}{d\tau^2} + \sum_{m,n=1}^{N,N}\Gamma^{(nm)\sigma}{}_{\mu\nu}\frac{d^n x^\mu}{d\tau^n}\frac{d^m x^\nu}{d\tau^m} = 0 \quad (23)$$

where the $\Gamma^{(nm)\sigma}{}_{\mu\nu}$'s are chosen to be symmetric in *nm*, and the first few are:[13]

$$\Gamma^{(11)\sigma}{}_{\mu\nu} \equiv \Gamma^\sigma_{\mu\nu} + \frac{g^{\sigma\rho}}{2}\left(\frac{d}{d\tau}\frac{\partial g_{\mu\nu}}{\partial\frac{dx^\rho}{d\tau}} - \frac{d^2}{d\tau^2}\frac{\partial g_{\mu\nu}}{\partial\frac{d^2x^\rho}{d\tau^2}} + \cdots\right) \quad (24)$$

$$\Gamma^{(12)\sigma}{}_{\mu\nu} \equiv \frac{g^{\sigma\rho}}{2}\left(\frac{\partial g_{\mu\nu}}{\partial\frac{dx^\rho}{d\tau}} + \frac{\partial g_{\nu\rho}}{\partial\frac{dx^\mu}{d\tau}} - 2\frac{d}{d\tau}\frac{\partial g_{\mu\nu}}{\partial\frac{d^2x^\rho}{d\tau^2}} + \cdots\right) \quad (25)$$

$$\Gamma^{(22)\sigma}{}_{\mu\nu} \equiv -g^{\sigma\rho}\frac{\partial g_{\mu\nu}}{\partial\frac{d^2x^\rho}{d\tau^2}} + \cdots, \quad (26)$$

$$\Gamma^{(13)\sigma}{}_{\mu\nu} \equiv \frac{g^{\sigma\rho}}{2}\left(\frac{\partial g_{\nu\rho}}{\partial\frac{d^2x^\mu}{d\tau^2}} - \frac{\partial g_{\mu\nu}}{\partial\frac{d^2x^\rho}{d\tau^2}}\right) + \cdots. \quad (27)$$

The only low-derivative expression here is the leading term in (24) which is just the Christoffel connection in GR,

$$\Gamma^\sigma_{\mu\nu} \equiv \frac{g^{\sigma\rho}}{2}\left(\frac{\partial g_{\nu\rho}}{\partial x^\mu} + \frac{\partial g_{\mu\rho}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\rho}\right). \quad (28)$$

This is the only term that survives when the metric $g_{\mu\nu}(x)$ depends only on spacetime, and one recovers from (23) the geodesic equation in Einstein's GR,

$$\frac{d^2x^\sigma}{d\tau^2} + \Gamma^\sigma_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0. \quad (29)$$

Thus, higher-derivative arguments in the metric lead to higher-derivative extensions of the Christoffel connection. One can think of the geodesic equation (23) as representing the inertial law in some "higher-derivative manifold" spanned by $d^n x^\mu/d\tau^n$ with $n = 0,1,\ldots,N$.

*D. Higher-derivative field equations*

In Einstein's theory of gravity, the geometry of the spacetime governs the dynamics of objects through the geodesic equation (29), while the distribution of matter in turn determines the curvature of the spacetime.[12] Since the law of inertia (23) involves higher-derivatives, the consequent unconventional motion and distribution of matter must determine some geometry described by higher-derivative field equations. We saw in the last section that higher-derivative arguments in the metric lead to higher-derivative extensions of the Christoffel connection. Consequently, the curvature tensor

$$R_{\mu\nu}{}^\rho{}_\sigma \equiv \frac{\partial\Gamma^\rho_{\nu\sigma}}{\partial x^\mu} - \frac{\partial\Gamma^\rho_{\mu\sigma}}{\partial x^\nu} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma} \quad (30)$$

shall likewise incur higher-derivative extensions. Since one always desires to sustain general covariance, the leading higher-derivative extensions must ultimately inhabit the forms of higher-order curvature invariants. Thus, the formal higher-derivative extension of general relativity may be represented by the Lagrangian,

$$\mathcal{L} = \sqrt{-g}\big(\lambda_0 + \lambda_1 R + \lambda_2 R^2 + \lambda_3 R_{\mu\nu}R^{\nu\mu} + \lambda_4 R_{\sigma\mu\nu\sigma}R^{\mu\nu} + \lambda_5 R_{\mu\nu}R^{\nu\sigma}R_{\sigma\mu} + \lambda_6 R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} + \cdots\big) \quad (31)$$

where $\lambda_i$'s are coupling constants. Einstein's GR comes as a special case of (31) with $\lambda_0 = -\Lambda/\kappa$, $\lambda_1 = \frac{1}{2\kappa}$ and $\lambda_{i>1} = 0$, where $\Lambda$ is the cosmological constant and $\kappa$ is Einstein's constant.

Theories based on higher-derivative invariants have actually been pursued in various contexts, particularly in the quest to subdue the infinities that plague the quantization of general relativity. For instance, the non-renormalizability of Einstein's theory of gravity in four dimensions[14] finds a cure if one includes the squares of the curvature in the Lagrangian [15-19],

$$\mathcal{L} = \sqrt{-g}\big(\lambda_0 + \lambda_1 R + \lambda_2 R^2 + \lambda_3 R_{\mu\nu}R^{\nu\mu}\big). \quad (32)$$

The calculation of higher-derivative counter-terms that hopes to render prominent theories finite continues to be pursued even at leading-loop approximations [20-22]. A recent example is the use of the divergent part of the pure Yang-Mills Lagrangian [23],

$$\mathcal{L}^{(1)}_{pYM}(D,4) = \frac{\hbar g^4}{2(4\pi)^{D/2}}\int_0^\infty ds\, s^{3-D/2} f^{abe} f^{bcf} f^{cdg} f^{dah} \left\{\frac{238+D}{360} F^e_{\mu\nu}F^f_{\nu\rho}F^g_{\rho\sigma}F^h_{\sigma\mu} + \frac{-50+D}{288} F^e_{\mu\nu}F^f_{\nu\mu}F^g_{\rho\sigma}F^h_{\sigma\rho}\right\} \quad (33)$$

in the quest to render supergravity amplitudes finite in *D*=8 dimensions [24]. In this non-Abelian theory, the field strength tensor is

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \quad (34)$$

where the structure constants define the Casimir of the adjoint representation,

$$C\delta^{cd} = f^{abc} f^{abd}. \quad (35)$$

*E. Higher-derivative field equations*

Assuming that higher-derivative arguments in the metric ultimately lead to a Lagrangian density that depends on the metric and its gradients, one may construct a higher-derivative geometrical theory of gravitation via a principle of least action. In the spirit of (31), one might start with a pure gravity action of the form

$$\int d^4x\mathcal{L} \equiv \int d^4x\sqrt{-g}\; G(g_{\mu\nu}, \partial g_{\mu\nu}/\partial x^\rho, \ldots) \quad (36)$$

where *G* is an arbitrary scalar that depends only on the geometry through $g_{\mu\nu}$ and its gradients. Requiring the action to be stationary under arbitrary continuous coordinate transformations,

$$x^\mu \to x^\mu + \delta x^\mu \quad (37)$$

yields the contracted Bianchi identities [25],

$$G^{\mu\nu}{}_{;\nu} = 0 \quad (38)$$

where the ";" indicates the covariant derivative and

$$G_{\mu\nu} \equiv \frac{1}{\sqrt{-g}}\left(\frac{\partial}{\partial g^{\mu\nu}} + \sum_{n=1}^{N}\left(\prod_{p=1}^{n}\frac{-\partial}{\partial x_{\alpha_p}}\right)\frac{\partial}{\partial\left(\prod_{p=1}^{n}\frac{\partial}{\partial x_{\alpha_p}}\right)g^{\mu\nu}}\right)\left(\sqrt{-g}G\right) \quad (39)$$

Sources proportional to the energy-momentum tensor $T^{\mu\nu}$ may be introduced so one can write the generalized field equation as

$$\frac{1}{\sqrt{-g}}\left(\frac{\partial}{\partial g^{\mu\nu}} + \sum_{n=1}^{N}\left(\prod_{p=1}^{n}\frac{-\partial}{\partial x_{\alpha_p}}\right)\frac{\partial}{\partial\left(\prod_{p=1}^{n}\frac{\partial}{\partial x_{\alpha_p}}\right)g^{\mu\nu}}\right)\left(\sqrt{-g}G\right) = -\kappa T_{\mu\nu} \quad (40)$$

Indeed, the humblest choice for *G* is the Ricci scalar *R* which leads to Einstein's general relativity. But (36) opens a portal to higher-derivative gravity theories. For instance, the Lagrangian based on quadratic invariants in the curvature tensor (32), yields the field equation [25]

$$-\tfrac{1}{2}g_{\mu\nu}\left(R^2 + \xi R_{\alpha\beta}R^{\alpha\beta}\right) + 2RR_{\mu\nu} + 2\xi R_{\mu\alpha\beta\nu}R^{\alpha\beta} + (2+\xi)R_{;\mu\nu} - \left(2+\tfrac{1}{2}\xi\right)g_{\mu\nu}\partial^\lambda\partial_\lambda R - \alpha\partial^\lambda\partial_\lambda R_{\mu\nu} = -\kappa T_{\mu\nu} \quad (41)$$

where $\xi$ is a dimensionless constant. As a candidate extension of general relativity, this must be made compatible with Newton's law of gravitation in the weak-field limit [25] and must somehow circumvent the Ostrogradskian instability [26] inherent in higher derivative theories.

## III. Conclusion

In the light of the recent resurgence of interest in higher-derivative theories, particularly in the quest to quantize and/or adapt gravity or the law of inertia to the issues of missing mass and accelerated expansion of the universe [27], it is illuminating to rediscover the roots of these theories from the observer's appreciation of time. We have demonstrated how the observer's operational assumptions about time promulgate dynamical laws. However, attempts to canonically quantize higher-derivative theories either lead to negative norm states ("ghosts"), or negative energy states that cause runaway particle production [28]. The classical origins of these problems were recognized a long time ago by Ostrogradski [26] who showed that if the Hamiltonian for a theory involving higher than the usual second derivative is obtained through Legendre transformations with respect to the derivatives of the field, the Hamiltonian incurs linear instabilities in its canonical momenta. Theories employing curvature invariants such as those in (31) or the Weyl invariant [29] $C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$, are beleaguered by Ostrogradski's ghost and negative energies. Any theoretical model that involves higher derivatives must therefore exorcise this ghost or provide an alternative interpretation of negative energies. This has led some to seriously consider nonlocal theories that depend on an infinite order of time-derivatives. Nonlocal theories have been studied, and are actually obtained as effective models of string theories, but their phenomenology is still beyond our brightest conquistadors. While some believe that this Ostrogradskian instability summatively precludes higher-derivative theories, one might appreciate it as a beacon that provides important clues that may guide those in search for the ultimate parent theory that accommodates all realms inquiry.

# References

[1] J.D. Anderson; J.K. Campbell; J.E. Ekelund; J. Ellis; J.F. Jordan (2008), "Anomalous Orbital-Energy Changes Observed during Spacecraft Flybys of Earth", Phys. Rev. Lett. 100 (091102), http://www2.phys.canterbury.ac.nz/editorial/Anderson2008.pdf

[2] J. D. Anderson et al., Phys. Rev. D 65, 082004 (2002), gr-qc/0104064

[3] M. M. Nieto and S. G. Turyshev, Class. Quant. Grav. 21, 4005 (2004), gr-qc/0308017

[4] S. G. Turyshev, V. T. Toth, L. R. Kellogg, E. L. Lau, and K. J. Lee, Int. J. Mod. Phys. D 15, 1 (2006), gr-qc/0512121

[5] Choi, C. Q. (March 3, 2008). "NASA Baffled by Unexplained Force Acting on Space Probes". Space.com. Retrieved 2011-02-12.

[6] Anderson, J.D., Laing, P.A., Lau, E.L., Liu, A.S., Nieto, M.M., and Turyshev, S.G., 1998

[7] M. Milgrom: Ap. J. 270, 365 (1983); M. Milgrom, MOND - theoretical aspects, New Astr. Rev., 46(741), 2002.

[8] J. M. Romero and A. Zamora, Alternative proposal to modified Newtonian dynamics, Phys. Rev. D, 73, (027301) (2006).

[9] D. N. Spergel et al. (WMAP), Astrophys. J. Suppl. 148, 175 (2003), astro-ph/0302209; D. N. Spergel et al. (WMAP), Astrophys. J. Suppl. 170, 377 (2007), astro-ph/0603449.

[10] Einstein, Albert; Podolsky, Boris and Rosen, Nathan (May 1935). "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?". Physical Review 47 (10): 777–780. Bibcode:1935PhRv...47..777E. doi:10.1103/PhysRev.47.777;

Aspect, Alain; Dalibard, Jean and Roger, Gérard (December 1982). "Experimental Test of Bell's Inequalities Using Time- Varying Analyzers". Physical Review Letters 49 (25): 1804–1807.

Bibcode:1982PhRvL..49.1804A. doi:10.1103/PhysRevLett.49.1804;

Jonathan Oppenheim; Stephanie Wehner (2010). "The uncertainty principle determines the non-locality of quantum mechanics". Science 330 (6007): 1072–1074. arXiv:1004.2507.

Bibcode:2010Sci...330.1072O. doi:10.1126/science.1192065. PMID 21097930; Marius Junge; Carlos Palazuelos (2010). "Large violation of Bell inequalities with low entanglement". arXiv:1007.3043v2 [quantph]; Thomas Vidick; Stephanie Wehner (2010). "More Non-locality with less Entanglement". arXiv:1011.5206v2 [quant-ph].

[11] R. Loll, Discrete Lorentzian Quantum Gravity, arXiv:hep-th/0011194v1 21 Nov 2000; J. Ambjørn, A. Dasgupta, J. Jurkiewicz, and R. Loll, A Lorentzian cure for Euclidean troubles, arXiv:hep-th/0201104 v1 14 Jan 2002

[12] C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation*, (W.H. Freeman & Co., San Francisco, 1973), 1279 pp; Ray A D Inverno, Introducing Einstein's Relativity, Publisher: Oxford University Press, USA ISBN: 0198596863, EAN:9780198596868, Publish Date: 199206-30.

[13] E. Rodulfo, E. Bornilla, The Manila Journal of Science 6: 2 (2011) pp. 19-29.

[14] G.'t Hooft and M. Veltman, Ann. Inst. H. Poincare, 20, 69 (1974).

[15] K. S. Stelle, Phys. Rev. D16, 953 (1977).

[16] W. Thirring, Phys. Rev. 77, 570 (1950).

[17] I. Antoniadis and E.T. Tomboulis. Phys. Rev. D99. 2756 (1986).

[18] S. Deser and P. van Nieuwenhuizen. Phys. Rev.Dl0, 401 (1974).

[19] E.T. Tomboulis. Phys. Rev. Lett. 52. 1173 (1984).

[20] E.S. Fradkin and A.A. Tseytlin, Nucl. Phys. B227, 252 (1983).

[21] A.E.M. van de Ven, Nucl. Phys. B250, 593 (1985).

[22] R.R. Metsaev and A.A. Tseytlin, Nucl. Phys. B298, 109 (1988).T. Tomboulis. Phys. Rev. Lett. 52. 1173 (1984).

[23] E. Rodulfo and R. Delbourgo, Int. J. Mod. Phys. A 14, 4457 (1999) [hep-th/9709201].

[24] Zvi Bern, Scott Davies, Tristan Dennen, Yu-tin Huang. (2012) Ultraviolet cancellations in half-maximal supergravity as a consequence of the double-copy structure. Physical Review D 86:10, Online publication date: 1-Nov-2012.

[25] A. J. Accioly, Revista Brasileira de Fisica, Vol. 18, N0. 4, 1988.

[26] M. Ostrogradski: Mem. Ac. St. Petersbourg VI 4, 385 (1850); F. J. de Urries, J. Julve, J. Phys. A31, 6949 (1998) (arXiv: hepth/9802115); F. J. de Urries, J. Julve, E. J. Sanches, J. Phys. A34, 8919 (2001) (arXiv: hepth/0105301).

[27] Alvaro Nunez, and Slava Solganik, Ghost constraints on modified gravity, Phys.Lett. B608 (2005) 189-193; Takeshi Chiba, Generalized gravity and ghost, JCAP 0503 (2005) 008; Gianluca Calcagni, Shinji Tsujikawa, and M. Sami, Dark energy and cosmological solutions in second-order string gravity, Class.Quant.Grav. 22 (2005) 3977-4006; Jacob Moldenhauer, Mustapha Ishak, John Thompson and Damien A. Easson, Supernova, baryon acoustic oscillations, and CMB surface distance constraints on f (G) higher order gravity models, Phys.Rev. D81 (2010) 063514; Timothy Clifton, Pedro G. Ferreira, Antonio Padilla, and Constantinos Skordis, Modified Gravity and Cosmology, Phys.Rept. 513 (2012) 1-189.

[28] A. Pais, G. E. Uhlenbeck, Phys. Rev. 79, 145 (1950); W. Heisenberg, Nucl. Phys. 4, 532 (1957).

[29] Philip D. Mannheim, Making the case for conformal gravity, Found.Phys. 42 (2012) 388-420; H. Lu, Y. Pang, C. N. Pope, and Justin F. Vazquez-Poritz, AdS and Lifshitz black holes in conformal and Einstein-Weyl gravities, Phys.Rev. D86 (2012) 044011.